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An Overview of Reliability Growth Models and Their Potential Use for NASA Applications

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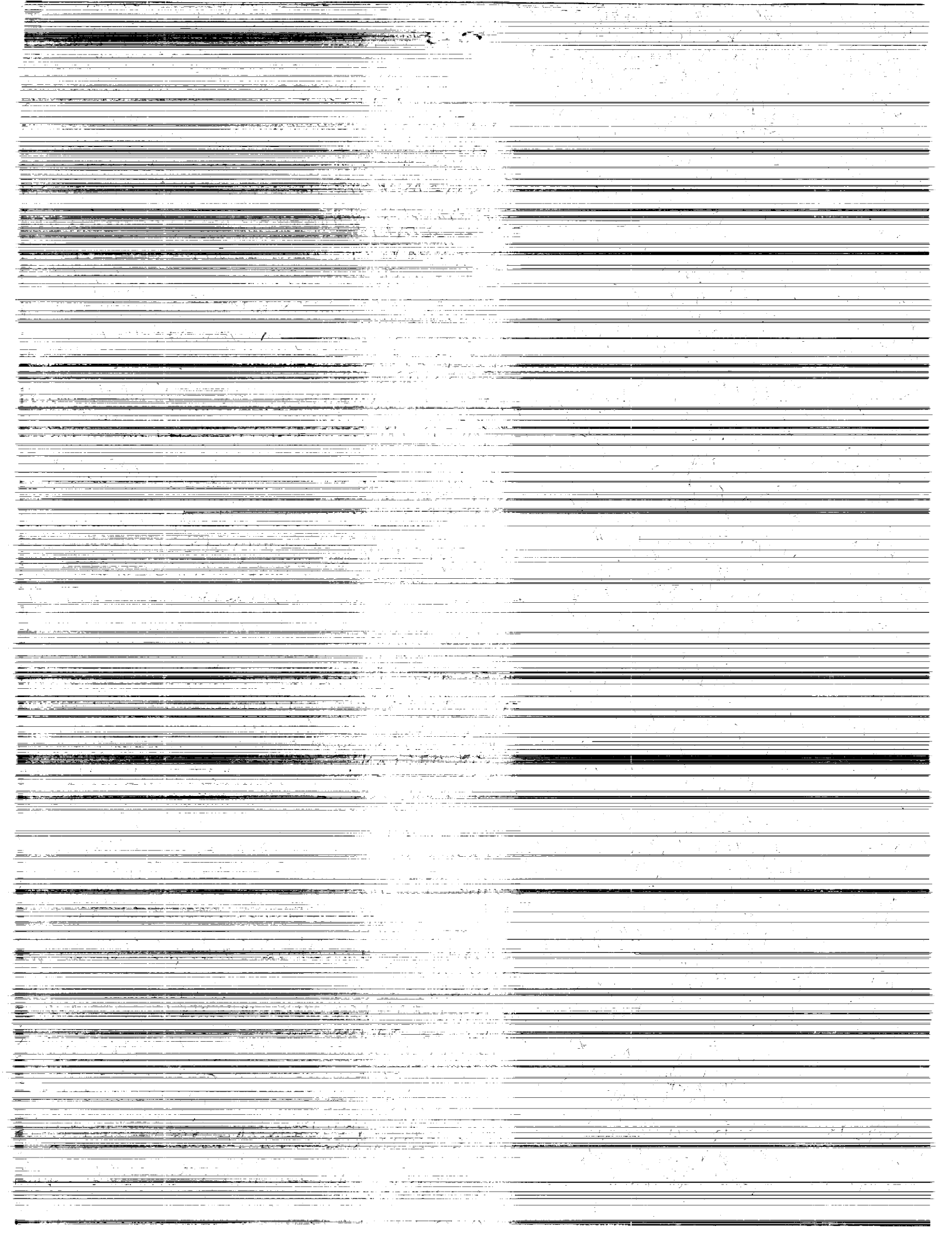
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Growth Models and
Their Potential Use
for NASA Applications**

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National Aeronautics and
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DEFINITION OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>
MTBF	mean time between failures
t	cumulative test time
$N(t)$	number of system failures by time t
TAAF	test, analyze, and fix
AMSAA	Army Materials System Activity Analysis
HPP	homogeneous Poisson process
NHPP	nonhomogeneous Poisson process
$\rho(t)$	intensity function of NHPP
MLE	maximum likelihood estimate
CI	confidence interval
SSME	space shuttle main engine
STME	space transportation main engine
LRU	line replaceable unit

TECHNICAL PAPER

AN OVERVIEW OF RELIABILITY GROWTH MODELS AND THEIR POTENTIAL USE FOR NASA APPLICATIONS

I. SUMMARY

This study provides an overview of reliability growth literature over the last 25 years. This includes a thorough literature review of the different areas of applications of reliability growth such as design, prediction, tracking/management, and demonstration. Various reliability growth models use a different basis on how they characterize growth. Different models are discussed in this report.

Also, this report addresses the use of reliability growth models to NASA applications. This includes the application of these models to the space shuttle main engine (SSME) growth process. For potential NASA applications, we classify growth models in two groups: growth models for management and growth models for demonstration. Both groups of models are characterized in this report.

II. INTRODUCTION

While quality control has always been of some concern in manufacturing ventures, only during the last two to three decades has reliability become of major concern to all parts of the engineering community. This is because complex modern designs carry within them the possibility of various types of error and malfunction.

Technology has been characterized in the past two decades by the development of complex systems containing a large number of subsystems, components, and parts. The trend to ever-larger and more complex systems is continuing with the development of space vehicles, weapons systems, communication systems, etc. Although the failure of a single inexpensive part or component may cause the failure of the entire system, reliability in general has not been considered as one of the system parameters such as cost or performance. It is very important to mention that unreliability has consequences in cost, idle time, psychological effects, and, in some cases, human life and national security.

In the design and manufacturing of a complex system, such as the space transportation main engine (STME), the initial prototypes will invariably have significant reliability and performance deficiencies. Consequently, such a system is subjected to development testing. When a failure occurs during development testing, the cause is isolated and a corrective action is implemented. Reliability growth is a method for quantifying and monitoring system reliability during the development process through the collection and analysis of the relevant data. Although generally not contractually required, the existence of a reliability growth program during early development phases increases the likelihood that more problems will be resolved earlier in the program, thus reducing large costs later.

Reliability growth is the positive improvement in reliability due to changes in product design or the manufacturing process.¹ It is expected that this process of finding problems and fixing them will

result in increasing mean time between failures (MTBF). This has led to the development of various statistical models. These models characterize growth in different ways. Some use failure rate as a function of time, some use MTBF, and others are based on the idea of success probability. In fact, any reliability characterization that is appropriate may be used, as long as the model development will provide a tool to quantify growth.

Any study of reliability growth has two main objectives. The first objective is the prediction of reliability at some future instant. In this case, the concern is to estimate rate of growth and to predict the achieved reliability up to the present time. This will help determine whether future reliability requirements will be met or not. In addition, the reliability growth study could provide information about growth patterns that might be useful to support decisions regarding the future of the program. Thus, one might investigate how growth of a system is related to such factors as number of tests, cost, design reviews, and number and types of design changes. For example, in the case of a liquid rocket engine, the reliability curve (reliability against number of tests) might "level off" even when reliability is relatively low. That is, we find that continued testing and test-analyze-and-fix (TAAF) has reached "a point of diminishing return." For all practical purposes, ultimate reliability of the hardware has been reached. If this value is not satisfactory, then a major design/process change might have to be introduced into the program.

The second objective of any reliability growth study is statistical inference and estimation of reliability for demonstration purposes. In this case, the main concern is to verify a reliability requirement at a specified confidence level. To do this, data from both tracking phase (development) and certification phase are used. Specifically, to verify a certain reliability requirement, a test plan is developed to track reliability during development and to demonstrate the reliability requirement by incorporating the system-level testing during certification.

Reliability managers have long been aware of the fact that the reliability of the system should improve as it progresses through development, but whether this growth will meet a targeted reliability is always a concern. To track reliability during development, a reliability growth management program can be used. This will furnish the manager with tools to evaluate and control program progress. A reliability growth management program will enable the manager to:

1. Take advantage of experience gained in previous programs
2. Evaluate different potential test plans and select the appropriate one
3. Evaluate possible course of failures and the appropriate corrective actions when an ongoing program is experiencing problems
4. Correctly evaluate the progress made by an ongoing program.

A constant cause of concern for the development engineer and management is that reliability demands an excessive number of tests for reliability demonstration. For example, if a program requirement calls for 0.99-percent reliability with 90-percent confidence for demonstration, then 230 tests with zero failure are required if a classical binomial model is used. Therefore, more innovative techniques need to be used. Reliability growth models are potential candidates for more efficient testing, which is the subject of this report.

III. OBJECTIVE

The objective of this study is to explore the various reliability growth models for reliability tracking and the potential applicability of these models to NASA programs.

To accomplish this task, the Redstone Scientific Information Center was used to compile a bibliography of 111 documents (books and articles) related to reliability growth. This literature review covers the past 25 years of work on reliability growth modeling, assessment, tracking, prediction, and control. In section V, a review of this literature is conducted, and models are then grouped into various categories for easy reference. Applications of reliability growth models are discussed in section VI. In section VII, some conclusions and recommendations are outlined.

IV. NEED FOR GROWTH MODELS

The development of a system often takes place by testing a system until it fails, then improving the design and testing again. At various times in this development process, it becomes important to assess and predict system reliability. The reliability growth of a system takes place due to changes introduced into the system structure. These changes make it difficult to estimate the system reliability using a classical model such as the binomial model. Also, classical models require the use of system test data for reliability demonstration. However, for some systems, test data are either not available or are available but very sparse. Thus, it is desirable to model the growth by a "growth model."

A reliability growth model is an analytical tool used to monitor the reliability progress during the developmental program, and to establish a test plan to demonstrate acceptable system reliability. The quantification provided by a growth model is valuable for proper management of a reliability program. Specifically, some of the advantages of using a reliability growth model are:⁷

1. Determining the intensity of TAAF to reach reliability objectives
2. Predicting whether stated reliability objectives will be achieved
3. Correlating reliability changes with reliability activities and tracking progress
4. Planning for a reliability demonstration test
5. Computing confidence limits to satisfy reliability requirements.

At present, there is no lack of available models for the reliability engineers. The main problem is a lack of guidance for the selection of the best model suitable for individual application. Various reliability growth models use a different basis to characterize growth. For example, some are based on times between failures, some count the number of failures, and others use a homogeneous or nonhomogeneous Poisson process. The process of choosing a model is not a simple one. Balaban,⁷ Barr,⁹ Gottfried,⁵² Jayachandran,⁶⁰ and Jewell⁶¹ have discussed model comparison. In some cases, this has aided in model selection. The information in this report is intended to help the reader in the selection and application of growth models.

V. OVERVIEW OF RELIABILITY GROWTH LITERATURE

This section involves a literature review of the various types of reliability growth models. A listing of the papers and books from the literature search is included in the bibliography. In section V.A, the bibliography is categorized by subject areas for easy reference. Section V.B discusses selected models which use failure rate or times between failures as a criterion to characterize growth; while in section V.C, selected models using success probability are discussed.

A. Bibliographical Characterization

An extensive search of the literature was conducted to identify important reliability growth articles and books, especially those dealing with recent developments in reliability growth management, tracking, demonstration, and related applications. These articles and books are listed in the bibliography. Additional important references are cited in the papers and books listed. Important articles and books are then categorized according to subject area for easy reference. The following is the list of the categories established.

	Reference:
Growth models	1, 7, 9, 17, 20, 24, 33, 39, 52, 58, 60, 66, 83, 101, 104
Tracking/management	8, 10, 16, 26, 27, 28, 30, 54, 64, 69, 75, 99, 109, 111
NASA applications	50, 94, 95, 105
Other applications	18, 19, 24, 26, 64
Reliability estimation	3, 25, 35, 36, 78, 81, 82, 107
Case studies	18, 19, 56
Prediction	14, 39, 40, 105
Design reliability	22, 38, 50, 88, 93, 94
Parameter estimation/CI	31, 43, 47, 68
Simulation/Monte Carlo	11, 41, 52, 108
Bayesian analysis	75, 76, 85, 91, 99, 104, 106
General theory	4, 22, 29, 41, 51, 61, 62, 67, 71, 74, 79, 80, 84, 98, 101.

B. MTBF Growth Models

Based on the literature review, selected models which use MTBF as a basis to characterize growth are discussed in this section. It should be noted that the U.S. Army Material Systems Analysis Activity (AMSAA) model has been given more attention in the discussion because of the wide applicability, mathematical simplicity, and completeness of the model development.

Duane's Model:

This model is one of the most widely used of all reliability growth models. Duane³⁹ used development data from several different systems, and concluded that the logarithm of observed cumulative mean time between failures (CMTBF) is a linear function of time. His model can be expressed as:

$$N(t) = Kt^{1-\alpha} , \quad (1)$$

where K is a constant, and α is the growth parameter. For this model, the cumulative failure rate $\lambda(t)$ is given by,

$$\lambda(t) = N(t)/t = Kt^{-\alpha} . \quad (2)$$

Also, from equation (1),

$$\mu(t) = K(1-\alpha)t^{-\alpha} = (1-\alpha) \cdot \lambda(t) . \quad (3)$$

This shows the current failure rate $\mu(t)$ is $(1-\alpha)$ times the cumulative failure rate, or taking reciprocals, the current MTBF = $1/(1-\alpha)$ times the CMTBF.

Duane suggested that during the development phase, his model will hold. In practice, from the available data, one takes test hours and test failures, and plots them on a log-log plot. The constant K and the growth parameter α are determined from the graph. This is Duane's postulate which is a deterministic learning curve formulation of reliability growth.

Duane's model predicts that the MTBF for the operational period following the completion of a development test of length T will be $MTBF(T) = [KT^{-\alpha}]^{-1}$. That is, the future MTBF is a function of the growth parameter and test length time T . A number of authors have suggested improvements on Duane's model since its publication in 1964, pointing out the limitations of the model. In particular, Finkelstein⁴⁸ claims that Duane's model is unsuited for planning since its boundary conditions do not reflect development testing experience. These boundary conditions are the MTBF at times $t = 0$ and $t = \infty$. At $t = 0$, $MTBF(0) = 0$, and in the limit as it gets large, $MTBF(\infty) = \infty$. Finkelstein modifies Duane's model by introducing an additional parameter in the model. Gottfried⁵² is also concerned about the behavior of Duane's model at $t = 0$, and that the model implies continuous reliability growth in time, which is inconsistent with the basic premise of TAAF—that growth occurs only when problems are recognized and corrected. From equation (1)

$$t = \left(\frac{N(t)}{K} \right)^{1/(1-\alpha)} \quad N(t) = 1, 2, 3, \dots . \quad (4)$$

By setting the mean time of occurrence of the first failure as follows:

$$E(t) = \theta = \left(\frac{1}{K}\right)^{1/(1-\alpha)},$$

Gottfried examines reliability growth with the use of a variant of Duane's model. This variant is based on stepwise growth which eliminates the above two problems with MTBF at $t = 0$ and $t = \infty$.

Duane's model can be used for detecting the presence of reliability trends. Codier²⁴ describes the use of Duane's growth curve to aid in determining the number of burn in cycles required to achieve a given failure rate for a radar LRU. For application of Duane's model to the reliability growth evaluations of several aircraft test programs such as the F-16 and the B-52, see Bower.¹⁹ Kasouf and Weiss⁶⁴ apply Duane's model to an integrated missile reliability growth program.

AMSAA Model:

The AMSAA model was developed by Crow and is reported in appendix C of MIL HDBK 189.¹ The AMSAA reliability growth model is designed for tracking the reliability within a test phase and not across tests phases. This model assumes that within a test phase, reliability growth can be modeled as a nonhomogeneous Poisson process (NHPP). This model, like Duane's model, also assumes that within a test phase, the cumulative failure rate is linear on log-log-scale. However, the AMSAA model is a local (within test phase) model, while Duane's model is a global (between phases) model. The AMSAA model evaluates the reliability growth that results from the introduction of design fixes into the system and not the reliability growth that may occur at the end of a test phase due to delayed fixes. For this model, Crow has developed rigorous statistical procedures useful for reliability growth tracking and demonstration.

Let t denote the cumulative test time from the beginning of the test phase. Let $0 < y_1 < y_2 < y_3 \dots < y_K$ denote the cumulative test times of design modifications on the system. Let λ_i denote the constant failure rate during the i th time period (y_{i-1}, y_i) between modifications at y_{i-1} and y_i . During development, more than one prototype is often tested. If the prototypes have the same basic configuration between corrections, then due to the constant failure rate assumption, the times y_i may be considered as cumulative test times on all prototypes up to i th correction. Also, on a cumulative time scale, N_i is the total number of failures for all systems during (y_{i-1}, y_i) .

Let $N(t)$ be the total number of system failures by time t , then,

$$E[N(t)] = \begin{cases} \lambda_1 t & \text{if } t \text{ is in the first interval,} \\ \lambda_1 y_1 + \lambda_2 (t - y_1) & \text{for } t \text{ in the second interval, and so on.} \end{cases}$$

Thus, $N(t)$ follows the NHPP⁸⁴ with mean value $\theta(t)$,

$$\theta(t) = \int_0^t \rho(y) dy, \quad (5)$$

where $\rho(x)$ is the intensity function given by,

$$\rho(x) = \lambda_i \text{ for } x \in (y_{i-1}, y_i), \quad i = 1, 2, \dots$$

The AMSAA model assumes that the intensity function can be approximated by a continuous parametric function, i.e.,

$$\rho(t) = \lambda \beta t^{\beta-1} \quad t > 0, \lambda > 0, \beta > 0, \quad (6)$$

which is the Weibull failure rate function. Thus, the mean value function is:

$$E(N(t)) = \theta(t) = \lambda t^\beta. \quad (7)$$

For $\beta = 1$, $\rho(t)$ is constant, indicating an homogeneous Poisson process (HPP). For $\beta < 1$, $\rho(t)$ is decreasing, indicating reliability growth. For $\beta > 1$, $\rho(t)$ is increasing, indicating deterioration in system reliability. It should be noted that the model assumes that $\rho(t)$ is approximated by a Weibull intensity function and not the Weibull distribution. Thus, statistical techniques used for the Weibull distribution are not applicable to $\rho(t)$.

The AMSAA model, like Duane's model, can also be used for detecting the presence of reliability trends through the evaluation of the model parameters and the instantaneous failure rate. However, the AMSAA model gives an analytical solution which provides estimates at specified confidence levels, while Duane's model provides a graphical solution which has limitations.

To estimate the parameters of the AMSAA model, two procedures exist. One procedure is used for time terminated testing, and the second one is for failure terminated testing.

Time Terminated Testing:

Using the maximum likelihood estimation (MLE), the shape parameter β is estimated as follows:

$$\hat{\beta} = \frac{N}{N \ln T - \sum_{i=1}^N \ln y_i}, \quad (8)$$

where N is the total number of failures up to test time T , and y_i are times of design modifications.

Using the MLE estimator of β , the estimator of λ is given by

$$\hat{\lambda} = \frac{N}{T^{\hat{\beta}}}. \quad (9)$$

Using the MLE of β and λ gives the estimate of $\rho(t)$,

$$\hat{\rho}(t) = \hat{\lambda} \hat{\beta} t^{\hat{\beta}-1}. \quad (10)$$

The reciprocal of $\hat{\rho}(t)$ provides an estimate of instantaneous MTBF. From equation (10), the estimate of $\hat{\rho}(t)$ can be written as:

$$\hat{\rho}(t) = \frac{N}{T^{\hat{\beta}}} \cdot \hat{\beta} \cdot T^{\hat{\beta}-1} = \frac{N}{T} \hat{\beta}.$$

Expressions (8), (9), and (10) provide point estimates for β , λ , and MTBF. To establish a confidence interval for the MTBF, the following expression is used,

$$\frac{L_{N,\tau}}{\hat{\rho}(T)} \leq \text{MTBF} \leq \frac{U_{N,\tau}}{\hat{\rho}(T)} , \quad (11)$$

where τ is the selected confidence level. $L_{N,\tau}$ and $U_{N,\tau}$ are available in a specially provided table available in reference 1. In addition to point and interval estimation, reference 1 provides a procedure for testing the model fitness using the Cramer-Von Mises goodness-of-fit test.

Failure Terminated Testing:

In this case, the MLE of β is given by,

$$\hat{\beta} = \frac{N}{(N-1) \ell_n y_N - \sum_{i=1}^{N-1} \ell_n y_i} , \quad (12)$$

where N and y_i are as defined in the time terminated testing case. The estimates of the scale parameter λ and the intensity function are given as above. A special table to compute interval estimates of MTBF has been provided in this case also. Again, Cramer-Von Mises statistics can be used to test goodness-of-fit between the model and the observed data. Statistical precision and robustness of the AMSAA model estimators are discussed by Ziad.¹¹¹ Wronka¹⁰⁹ applies the AMSAA model to grouped data to demonstrate reliability growth at an early development phase.

The AMSAA reliability growth model is a fully developed model which has been widely used in various areas. To list a few applications: Crow³⁰ applies the AMSAA model for tracking reliability during the development phase of army programs. Benton and Crow¹² give an application to an integrated reliability growth program using different types of testing (such as early prototype, environmental, safety, TAAF, reliability demonstration, etc.). Ellis and Gibson⁴² apply the AMSAA model to analyze repair times (the times required to complete maintenance actions). Safie⁹⁵ applies the AMSAA model to evaluate SSME reliability. It should be noted that this model, like Duane's model, is sensitive to an early initial failure.

IBM Model:

This model is based on the solution of differential equations generated using the following two assumptions:⁷

1. Two types of failures are possible: random failures occurring at rate λ (constant intensity function), and nonrandom failures due to design weakness defects. The number K of nonrandom failures is fixed but unknown at the beginning of testing.

2. If $M(t)$ is the number of nonrandom failures remaining at time t , the rate of change of $M(t)$ is proportional to $M(t)$. This implies:

$$\frac{dM(t)}{dt} = -K_2 M(t) , \quad (13)$$

and hence

$$M(t) = K_1 e^{-K_2 t} \quad K_1, K_2 > 0, \quad (14)$$

where K_1 is the initial number of failures.

An interesting feature of the IBM model is the ability of the model to predict the time when the system/equipment is “ q ” fraction debugged. In fact, from equation (14),

$$q = 1 - e^{-K_2 t}, \quad (15)$$

thus, having estimated K_2 , say \hat{K}_2 , we can find the time at which $q = 0.90$ of the nonrandom defects have been removed by solving equation (15). The solution is, in general,

$$t_{0.90} = \frac{-\ln(0.10)}{\hat{K}_2}.$$

Equation (15) is a powerful tool because it can be used to determine the length of development testing if K_2 has already been estimated.

Considering both types of failures as defined in the two assumptions, the expected cumulative number of failures up to time t is given by $V(t)$,

$$V(t) = \lambda t + K_1 - M(t) = \lambda t + K_1(1 - e^{-K_2 t}). \quad (16)$$

Using equation (16)

$$\text{Cum MTBF} = t/(\lambda t + K_1(1 - e^{-K_2 t})).$$

An unattractive feature of the IBM model is that equation (16) is nonlinear. Thus, the estimation of λ , K_1 , and K_2 should be accomplished by iterative methods.

Lloyd and Lipow Model:

This model, like the IBM model, is based on the solution of a differential equation. Let $\phi(t)$ denote the mean time between failures. Lloyd and Lipow⁷¹ propose a model in which the growth rate is inversely proportional to the square of time, i.e.,

$$\frac{d\phi(t)}{dt} = K_1/t^2, \quad K_1 > 0. \quad (17)$$

Using equation (17), $\phi(t)$ is given by

$$\phi(t) = \begin{cases} K_2 - K_1/t & t > K_1/K_2 \\ 0 & 0 < t < K_1/K_2 \end{cases}, \quad (18)$$

where K_2 is the constant of integration. As equation (18) indicated, MTBF is 0 for time period $(0, K_1/K_2)$ and the limiting MTBF is K_2 . To estimate the growth parameter K_1 , let $t' = 1/t$, then $\phi(t') = K_2 - K_1 t' \cdot \phi(t')$ is linear in t' with slope K_1 and intercept K_2 . Both the parameters K_1 and K_2 can be estimated by the usual least-squares method.

Aroef Model:

Like the two previous models, this model is based on the solution of a differential equation. The Aroef model (reported in reference 7) considers that growth rate is directly proportional to the growth parameters and inversely proportional to t^2 , i.e.,

$$\frac{d\phi(t)}{dt} = K_1 \phi(t)/t^2, \quad (19)$$

solving

$$\phi(t) = K_2 e^{-K_1/t}. \quad (20)$$

K_2 is the limiting MTBF since $\lim_{t \rightarrow \infty} \phi(t) = K_2$. Also $\lim_{t \rightarrow 0} \phi(t) = 0$, the initial MTBF, is 0. As in the previous model, the nonlinear form in equation (20) can be transformed to a simple linear equation and the least-squares method can be used to estimate the model parameters.

ARINC Research Model:

The ARINC research model proposed by Balaban⁷ is described as

$$\phi(0,t) = KG(t), \quad (21)$$

where K is the initial MTBF, $\phi(0,t)$ is the MTBF over the interval $(0,t)$, and $G(t)$ is the growth function defined as,

$$G(t) = M - (M-1) \exp(-\alpha t^\beta). \quad (22)$$

As indicated in equations (21) and (22), this model has four parameters. Balaban suggests that K can be estimated by the first observed MTBF. Thus, in practice, only three parameters need to be estimated. As indicated in the following equation

$$\ln \ln \left(\frac{M-1}{M-G(t)} \right) = \ln \alpha + \beta \ln t, \quad (23)$$

the parameters of the model are determined by the following procedure:

Step 1. Use the first applicable data point to estimate K .

Step 2. For each observation period, calculate $G(t) = \phi(0,t)/K$.

Step 3. Choose a value of M (the maximum growth rate), then use the value of $G(t)$ with the selected M to estimate α and β using the least-squares method.

Step 4. Repeat step 3 for different values of M to improve the fit.

Step 5. The best fit of step 4 yields least-squares estimates of α and β from which the future growth function can be calculated as

$$\hat{G}(t) = \hat{M} - (\hat{M} - 1) \exp(-\hat{\alpha} t^{\hat{\beta}}) .$$

To conclude this section, although only six models are discussed in section V.B, many other models using MTBF have been developed. Very few of these models have been widely used due to the difficulty involved in the estimation of the model parameters, the amount of subjectivity required to solve these models, and the lack of applicability of the model to real-life situations. It is important to emphasize that the AMSAA and the Duane's models are the most commonly used models in reliability growth applications.

C. Success Probability Models

In this section, we discuss reliability growth models which use mission success probability as the basis to characterize growth.

Lloyd and Lipow Models:

In reference 71, Lloyd and Lipow discuss a simple model for a system with one failure mode. Occasionally, a situation is experienced in a development program where a particular component repeatedly fails. If the system operates successfully for a given test, no corrective action is performed prior to the next test. However, if the system fails a corrective action is performed. Let α be the success probability of correcting the problem in the following test and p be the inherent probability of failure. Also, assume that any redesign effort either is completely successful or has no effect on the inherent failure probability p .

Let $p_n(0)$ denote the probability that the failure mode has been eliminated before the n th test and $p_n(1)$ denote the probability that the failure mode has not been eliminated before the next test, then

$$p_n(1) = p_{n-1}(1)(1-p) + p_{n-1}(1) \cdot p(1-\alpha) . \quad (24)$$

Lloyd and Lipow assume that $p_1(1) = 1$ and show that system reliability on the j th test is

$$R_j = 1 - (1 - p\alpha)^{j-1} \cdot p .$$

However, if $p_1(1) = \beta$, where $\beta < 1$, then R_j can be expressed as

$$R_j = 1 - A e^{-C(j-1)} , \quad (25)$$

where the parameters A and C are given by

$$A = \beta p$$

$$C = \log(1/(1-p\alpha)) > 0 .$$

Methods for estimating parameters A and C are discussed in reference 71.

Lloyd and Lipow also consider a growth model in which they assume that a test program is conducted in N stages. At each stage, a certain number of tests are made and the number of failures recorded. The results of each stage are used to improve the items for the next stage. The authors use the growth model:

$$R_j = R_\infty - \alpha/j , \quad (26)$$

where R_j is the reliability during the j th stage of testing, and R_∞ is the maximum reliability as $j \rightarrow \infty$. R_∞ and α are the model parameters. Let n_j = sample size for the j th test, s_j = number of successes on j th test,

$$\bar{n} = \frac{1}{N} \sum_{j=1}^N n_j \quad \text{and} \quad c_i = \sum_{j=1}^N \frac{1}{j} ,$$

then initial estimates of R_∞ and α are obtained by solving:

$$\frac{1}{\bar{n}} \sum_{j=1}^N j S_j = \frac{N(N+1)}{2} R_\infty - \alpha \cdot N ,$$

and

$$\frac{1}{\bar{n}} \sum S_j = N \cdot R_\infty - \alpha \cdot c_1 .$$

If \hat{R}_∞ and $\hat{\alpha}$ represent estimates after a given iteration, the next pair of estimates are obtained from the following equations:

$$\sum_{j=1}^N \frac{j S_j}{n_j D_j} = \left(\sum_{j=1}^N \frac{j}{D_j} \right) R_\infty - \left(\sum_{j=1}^N \frac{1}{D_j} \right) \cdot \alpha ,$$

$$\sum_{j=1}^N \left(\frac{S_j}{n_j D_j} \right) = \left(\sum_{j=1}^N \frac{1}{D_j} \right) R_\infty - \left(\sum_{j=1}^N \frac{1}{j D_j} \right) \cdot \alpha ,$$

where

$$D_j = \frac{j}{\bar{n}_j} (\hat{R}_\infty - \hat{\alpha}/K)(1 - \hat{R}_\infty + \hat{\alpha}/K) .$$

In reference 71, Lloyd and Lipow also give a lower confidence bound for R_j .

Wolman Model:

Wolman¹¹⁰ considers a model with two types of failures:

- Type I: Inherent Failures
- Type II: Design Weakness Failures.

He assumes that a number of type II failures are known, and, once a design weakness failure is eliminated, it will never again cause a system failure. He further assumes that the probability of a transient (system) failure may never increase due to corrective action, and the two types of failures occur independently. Let $q_o = P$ (inherent failure), and $q_i = P$ (design weakness failure, the system has experienced i failures which have been corrected), then

$$q_i = \left(\prod_{j=0}^i \beta_j \right) q, \quad 0 < \beta_j < 1 \quad j = 1, 2, \dots, i, \quad (27)$$

where q is the initial probability of design weakness failure, β_j is unknown, and $(1-\beta_j)$ represents a reduction in design failure probability by the j th corrective action with $\beta_0 = 1$. Let $P_n = P$ (success after n failures are observed), then under this model

$$P_n = 1 - q_0 - q_{n-1} = 1 - q_0 - q\beta^{n-1} \quad \text{if } \beta_j = \beta \text{ (Wolman model)} . \quad (28)$$

Wolman is interested in calculating quantities such as the probability of eliminating all K design weaknesses in $m \geq K$ trials. It is important to mention that all parameters are assumed known, and, as a result, no estimations are involved.

Barlow and Scheur Model:

Barlow and Scheur⁸ discuss a model for reliability growth assuming two types of failures as in the previous model. As in the Lloyd and Lipow test, the program is conducted in N stages. At each stage of experimentation, tests are run on similar items. The results of each stage of testing are used to improve items for further testing in the next stage. For the j th stage, a_j , b_j , and c_j , which represent the number of inherent failures, the number of design weakness failures and the number of successes, respectively, are recorded. Let $q_0 = P$ (inherent failure), $q_j = P$ (failure due to design weakness in the j th stage), and $r_j = P$ (of success in j th stage), then r_j , the reliability in j th stage, is given by

$$r_j = 1 - q_0 - q_j . \quad (29)$$

Assuming that the sequence of q_j 's is nonincreasing, i.e., the increase in r_j from stage to stage is accomplished by a decrease in q_j which is the result of the appropriate corrective actions, the MLE of q_0 and q_j are given by

$$\hat{q}_0 = \sum_{j=1}^N a_j / \sum_{j=1}^N (a_j + b_j + c_j) , \quad (30)$$

$$\hat{q}_j = (1 - \hat{q}_0) b_j / (b_j + c_j) \quad j = 1, 2, \dots, N . \quad (31)$$

These are MLE if $\hat{q}_1 \geq \hat{q}_2 \geq \dots \geq \hat{q}_N$. However, if $\hat{q}_j < \hat{q}_{j+1}$ for some j ($j = 1, 2, 3, \dots, N-1$), then combine the j th and the $(j+1)$ st stage and compute the MLE of q_j 's using equation (31) for the $(N-1)$ stages thus formed. This procedure is continued until the estimates of the q_j 's form a nonincreasing sequence. The MLE of the system reliability at the j th stage is

$$\hat{r}_j = 1 - \hat{q}_j - \hat{q}_0 . \quad (32)$$

A lower confidence bound for r_j is discussed in reference 8.

Virene Model:

Virene¹⁰⁵ considers the use of a Gompertz equation for reliability growth. His model has the following form:

$$R_t = ab^{c^t} \quad 0 < b < 1, \quad 0 < c < 1,$$

where R_t = reliability at stage t , a = upper limit approached by reliability as $t \rightarrow \infty$, and b and c are the other model parameters. A procedure to estimate these parameters is outlined in reference 105. Using the outlined procedure, Virene estimates a , b , and c by fitting observed data from the lunar orbiter spacecraft and Blue Scout launch vehicle. In his paper, Virene also provides a method to establish a lower confidence interval on reliability.

Bonis Model:

This model has the form

$$R_j = R_{\infty} - q\alpha^{j-1},$$

where

R_j = reliability on j th test

R_{∞} = ultimate reliability

q = initial reliability

α = reliability growth factor, is the ratio of unreliability at the end of a stage to that at the end of the previous stage.

Bonis²⁷ describes how his model can be used to determine reliability levels at each stage that will yield a final reliability consistent with an established goal. As test stage reliability data accumulate, the model is used to quantify actual progress, and this quantification of the data is necessary for proper reliability management.

In conclusion, various reliability growth models which use success probability to characterize growth exist. Some of the models might have potential use for NASA applications such as the Lloyd and Lipow model and others as discussed.

VI. APPLICATION OF GROWTH MODELS TO NASA PROGRAMS

For NASA applications, the growth models can be classified into two groups: models for reliability management and models for reliability demonstration. The main difference between the two groups of models is that the one used for demonstration emphasizes the model accuracy and goodness of fit, while the other emphasizes analysis and tracking. While the SSME program is using growth models

for reliability demonstration, the STME program is planning to use reliability growth models for reliability management. This section provides an SSME example application of the reliability growth models for reliability demonstration purposes.

Before going into the details of the SSME application, the following is a description of the process used to apply the AMSAA model to the SSME. Recalling from section V.B that the AMSAA model assumes that system failures follow an NHPP with Weibull intensity function given by,

$$\rho(t) = \lambda \beta t^{\beta-1} , \quad (33)$$

where λ is the scale parameter, and β is the shape parameter. β is estimated by

$$\hat{\beta} = \frac{N}{N \ln T - \sum_{i=1}^N \ln x_i} , \quad (34)$$

where N = number of failures, T = accumulated test time, and x_i = failure times for failures $i = 1, 2, \dots, N$. Using $\hat{\beta}$, the estimate of λ is

$$\hat{\lambda} = N/T^{\hat{\beta}} . \quad (35)$$

Using both parameter estimates, the instantaneous failure rate is given by

$$m(T) = 1/\hat{\rho}(T) ,$$

where

$$\hat{\rho}(T) = \hat{\lambda} \hat{\beta} T^{(\hat{\beta}-1)} . \quad (36)$$

From equation (36), the lower confidence limit for MTBF is

$$\frac{L_{N,\gamma}}{\hat{\rho}(T)} \leq \text{MTBF} , \quad (37)$$

where $L_{N,\gamma}$ is the table value for N and γ , the desired confidence level (reference 1, table C1).

The test of the null hypothesis that data fit the AMSAA model can be performed using the Cramer-Von Mises statistic C_N^2 ,

$$C_N^2 = \frac{1}{12N} + \sum_{i=1}^N \left[\left(\frac{x_i}{T} \right)^{\bar{\beta}} - \frac{2i-1}{2N} \right]^2 . \quad (38)$$

where

$$\bar{\beta} = \frac{N-1}{N} \hat{\beta} .$$

The null hypothesis is rejected if $C_N^2 > \text{critical value}$. These critical values are available in reference 1 for given n and α , the significance level.

Applying the above procedure for SSME data (the SSME data are not up-to-date and are used for illustration purposes only), given the following SSME cumulative failure times,⁹²

Failure (i)	Cumulative Failure Times, x_i , in seconds
1	505
2	10,348
3	10,872
4	15,516
5	15,844
6	48,168
7	48,476
8	55,606
9	78,724
10	97,648
11	158,674
12	206,712
13	270,242

and the total time of $T = 373,868$ s. The $\hat{\beta}$ and $\hat{\lambda}$ are determined using equations (34) and (35):

$$\hat{\beta} = 0.4228 \quad \text{and} \quad \hat{\lambda} = 0.05725 .$$

Notice that a β of 0.4278 indicates a growth. Using these $\hat{\lambda}$ and $\hat{\beta}$, the instantaneous MTBF is 68,021. The MTBF is then used to calculate the engine reliability for a mission time of 520 s,

$$R(520) = e^{-\text{mission duration}/\text{inst. MTBF}} = 0.9924 .$$

As described earlier, a lower bound on reliability can be established and testing the model goodness-of-fit can be performed using equations (37) and (38).

VII. CONCLUSIONS

In this report, we have provided a thorough review of the reliability growth literature with a complete bibliography of the materials searched. Also, we have identified and discussed some selected models such as the AMSAA, Duane, IBM, ARINC Research, Lloyd and Lipow, Barlow and Scheur, etc., models. Some of these models have been used for NASA applications. Other models might have potential use in future NASA programs.

As a concluding remark, it is important to keep in mind that using reliability growth during development and certification will provide a tool to track and improve reliability, enhance the effectiveness of a test program, and evaluate the effectiveness of design process changes.

BIBLIOGRAPHY

1. MIL-HDBK-189: "Reliability Growth Management." Available from Headquarters, U.S. Army Communication Research and Development Command, Fort Monmouth, NJ, February 1981.
2. Ascher, H.E.: "Evaluation of Repairable System Reliability Using Bad-as-Old Concepts." IEEE Trans. Reliability, vol. R-17, June 1968, pp. 103-110.
3. Ascher, H.E.: "Distribution-Free Estimation of System Reliability." Proc. Annual Reliability and Maintainability Symp., 1980, pp. 374-378.
4. Ascher, H.E.: "Weibull Distribution Versus Weibull Process." Proc. Annual Reliability and Maintainability Symp., 1981, pp. 426-431.
5. Ascher, H.E., and Feingold, H.: "Bad-as-Old Analysis of System Failure Data." Proc. Annual Reliability and Maintainability Conference, 1989, pp. 49-62.
6. Ascher, H.E., and Feingold, H.: "Is There Repair After Failure?" Proc. Annual Reliability and Maintainability Symp., 1978, pp. 190-197.
7. Balaban, H.S.: "Reliability Growth Models." J. Environmental Sciences, vol. 21, January-February 1978, pp. 11-18.
8. Barlow, R.E., and Scheuer, E.M.: "Reliability Growth During a Development Testing Program." Technometrics, vol. 8, February 1966, pp. 53-60.
9. Barr, D.R.: "Class of General Reliability Growth Models." Operations Research, vol. 18, 1970, pp. 52-65.
10. Bell, R.: "Army Reliability Growth Management Policy." Proc. Annual Reliability and Maintainability Symp., 1986, pp. 210-213.
11. Benski, H.C., and Cabau, E.: "Experimental Design Techniques in Reliability Growth Assessment." Proc. Annual Reliability and Maintainability Symp., 1992, pp. 322-326.
12. Benton, A.W., and Crow, L.H.: "Integrated Reliability Growth Testing." Proc. Annual Reliability and Maintainability Symp., 1989, pp. 160-166.
13. Bezat, A., and Montague, L.: "The Effect of Endless Burn-in on Reliability Growth Projections." Proc. Annual Reliability and Maintainability Symp., 1979, pp. 392-397.
14. Bezat, A., Norquist, V., and Montague, L.: "Growth Modeling Improves Reliability Predictions." Proc. Annual Reliability and Maintainability Symp., 1975, pp. 317-322.
15. Bhattacharya, G.K., Fries, A., and Johnson, R.A.: "Properties of Continuous Analog Estimators for a Discrete Reliability Growth Model." IEEE Transactions on Reliability, vol. 38, No. 3, August 1989.

16. Bieda, J.: "Reliability Growth Test Management in a Product Assurance Environment With the Automotive Component Industry." Proc. Annual Reliability and Maintainability Symp., 1991, pp. 317-321.
17. Bonis, A.J.: "Reliability Growth Curves for One Shot Devices." Proc. Annual Reliability and Maintainability Symp., 1977, pp. 181-185.
18. Bonnett, R.W., and Oglesby, R.D.: "Patriot Ground Equipment Reliability Growth." Reliability Growth Management, Testing, and Modeling, 1978, pp. 23-27.
19. Bower, J.N.: "Reliability Growth During Flight Test." Proc. Annual Reliability and Maintainability Symp., 1986, pp. 101-105.
20. Bresenham, J.E.: "Reliability Growth Models." Annual Technical Conference Transactions, ASQC, 1966, pp. 179-187.
21. Byers, J.K., and Galli, D.H.: "Reliability Growth Apportionment." IEEE Trans. Reliability, vol. 26, October 1977, pp. 242-244,
22. Chenoweth, H.B.: "A Theory of Reliability Growth in Structural Systems." Proc. Annual Reliability and Maintainability Symp., 1980, pp. 106-111.
23. Clarke, J.M.: "No-Growth Growth Curves." Proc. Annual Reliability and Maintainability Symp., 1979, pp. 407-410.
24. Codier, E.O.: "Reliability Growth in Real Life." Proc. Annual Reliability and Maintainability Symp., 1968, pp. 458-469.
25. Corcoran, W.J., Weingarten, H., and Zehna, P.W.: "Estimating Reliability After Corrective Actions." Management Science, vol. 10, July 1964, pp. 786-795.
26. Cox, T.D., and Kelley, J.: "Reliability Growth Management of SATCOM Terminals." Proc. Annual Reliability and Maintainability Symp., 1976, pp. 218-233.
27. Cougan, W.P., and Kindig, P.W.: "A Real-Life MTBF Growth Program for a Deployed Radar." Proc. Annual Reliability and Maintainability Symp., 1979, pp. 121-127.
28. Crow, L.H.: "Methods for Assessing Reliability Growth Potential." Proc. Annual Reliability and Maintainability Symp., 1984, pp. 484-489.
29. Crow, L.H.: "Reliability Analysis for Complex Repairable Systems." Soc. Industrial and Applied Mathematics, Reliability and Biometry, Proceedings of Statistical Analysis of Life Length, July 1974, pp. 379-410.
30. Crow, L.H.: "On Tracking Reliability Growth." Proc. Annual Reliability and Maintainability Symp., 1975, pp. 438-443.
31. Crow, L.H.: "Confidence Interval Procedures for the Weibull Process With Applications to Reliability Growth." Technometrics, vol. 24, February 1983, pp. 67-72.

32. Crow, L.H.: "Reliability Values in the Development Process." Proc. Annual Reliability and Maintainability Symp., 1985, pp. 383–387.
33. Crow, L.H.: "On the Initial System Reliability." Proc. Annual Reliability and Maintainability Symp., 1986, pp. 115–119.
34. Crow, L.H.: "New International Standards on Reliability Growth." Proc. Annual Reliability and Maintainability Symp., 1991, pp. 478–480.
35. Crow, L.H., and Basu, A.: "Reliability Growth Estimation With Missing Data—II." Proc. Annual Reliability and Maintainability Symp., 1988, pp. 248–253.
36. Dahiya, R.C.: "Estimation of Reliability After Corrective Action." IEEE Trans. Reliability, vol. R-26, December 1977, pp. 348–351.
37. Dhillon, B.S.: "Reliability Growth: A Survey." Microelectronics and Reliability, vol. 20, 1980, pp. 743–751.
38. Dhillon, B.S.: "Mechanical Reliability Interference Theory Models." Proc. Annual Reliability and Maintainability Symp., 1980, pp. 462–467.
39. Duane, J.T.: "Learning Curve Approach to Reliability Monitoring." IEEE Trans. Aerospace, vol. 21, 1964, pp. 563–566.
40. Earles, D.R.: "Reliability Growth Prediction During Initial Design Analysis." Seventh National Symposium on Reliability and Quality Control, 1964, pp. 563–566.
41. Ellis, K.E.: "Robustness of Reliability Growth Analysis Techniques." Proc. Annual Reliability and Maintainability Symp., 1992, pp. 303–315.
42. Ellis, K.E., and Gibson, G.J.: "Trend Analysis of Repair Times." Proc. Annual Reliability and Maintainability Symp., 1991, pp. 85–92.
43. Englewood, M., and Bain, L.J.: "Prediction Intervals for the Weibull Process." Technometrics, vol. 20, 1978, pp. 167–169.
44. Evans, R.A.: "Models for Reliability of Repaired Equipment." IEEE Trans. Reliability, vol. 19, November 1990, p. 127.
45. Ferting, F.W., and Murthy, V.K.: "Models for Reliability Growth During Burn-in." Proc. Annual Reliability and Maintainability Symp., 1978, pp. 504–509.
46. Fink, R.W.: "Screening for Reliability Growth." Proc. Annual Reliability and Maintainability Symp., 1971, pp. 316–320.
47. Finkelstein, J.M.: "Confidence Bounds on the Parameters of the Weibull Process." Technometrics, vol. 18, 1976, pp. 115–117.

48. Finkelstein, J.M.: "Starting and Limiting Values for Reliability Growth." IEEE Trans. Reliability, vol. R-28, June 1979, pp. 111-114.
49. Finkelstein, J.M.: "A Logarithmic Reliability-Growth Model for Single-Mission Systems." IEEE Transactions on Reliability, vol. R-32, 1983, pp. 508-511.
50. Fox, E.P., and Safie, F.M.: "Statistical Characterization of Life Drivers for a Probabilistic Design Analysis." AIAA-92-3414, AIAA/SAE/ASME/ASEE 28th Joint Propulsion Conference and Exhibit, 1992.
51. Gnedenko, B.V., Belyaev, Y.K., and Solovyew, A.D.: "Mathematical Methods of Reliability Theory." Academic, New York, 1969.
52. Gottfried, P.: "Some Aspects of Reliability Growth." IEEE Transactions on Reliability, vol. R-36, 1987, pp. 11-16.
53. Gray, C.T.: "A Modeling Framework for Reliability Growth." Proc. Annual Reliability and Maintainability Symp., 1986, pp. 13-20.
54. Gross, A.J., and Kamins, M.: "Reliability Assessment in the Presence of Reliability Growth." Annual Symposium on Reliability, 1968, pp. 406-416.
55. Green, J.E.: "The Problems of Reliability Growth and Demonstration With Military Electronics." Microelectronics and Reliability, vol. 12, 1973, pp. 513-520.
56. Hadel, J.J.: "The Eagle—A Classic Study in Reliability Growth." Proc. Annual Reliability and Maintainability Symp., 1984, pp. 476-483.
57. Hamilton, D.O., and Ness, W.G.: "Aircraft Reliability Growth Characteristics." Proc. Annual Symposium on Reliability, 1969, pp. 465-471.
58. Healy, J.: "A Simple Procedure for Reliability Growth Modeling." Proc. Annual Reliability and Maintainability Symp., 1987, pp. 171-175.
59. Heimann, D.I., and Clark, W.D.: "Process-Related Reliability-Growth Modeling—How and Why." Proc. Annual Reliability and Maintainability Symp., 1992, pp. 316-321.
60. Jayachandran, T., and Moore, L.R.: "A Comparison of Reliability Growth Models." IEEE Trans. Reliability, vol. R-25, April 1976, pp. 49-51.
61. Jewell, W.S.: "A General Framework for Learning Curve Reliability Growth Models." Operations Research, vol. 32, 1984, pp. 547-558.
62. Kapur, K.C., and Lamberson, L.R.: "Reliability in Engineering Design." John Wiley and Sons, Inc., New York, 1977.
63. Kasouf, G.: "The Fallacious Reliability Demonstration Test." Proc. Annual Reliability and Maintainability Symp., 1979, pp. 374-380.

64. Kasouf, G.: "An Integrated Missile Reliability Growth Program." Proc. Annual Reliability and Maintainability Symp., 1984, pp. 465-470.
65. Koo, D.Y.: "Growth Test Time—Key to Effective Test Planning." Proc. Annual Reliability and Maintainability Symp., 1981, pp. 389-394.
66. Kreuze, F.: "Growth Curves—A Practical Management Tool." Proc. Annual Reliability and Maintainability Symp., 1972, pp. 430-436.
67. Lawless, J.F.: "Statistical Models and Methods for Lifetime Data." John Wiley and Sons, Inc., New York, 1982.
68. Lee, L., and Lee, S.K.: "Some Results on Inferences for the Weibull Process." Technometrics, vol. 20, 1978, pp. 41-45.
69. Lilius, W.A.: "Reliability Growth Planning and Control." Proc. Annual Reliability and Maintainability Symp., 1978, pp. 267-270.
70. Littlewood, B.: "A Critique of the Jelinski-Moranda Model for Software Reliability." Proc. Annual Reliability and Maintainability Symp., 1981, pp. 357-364.
71. Lloyd, D.K., and Lipow, M.: "Reliability: Management, Methods, and Mathematics." Published by American Society for Quality Control, 1984.
72. Lloyd, F.H.: "Reliability Demonstration in a Transit Environment." Proc. Annual Reliability and Maintainability Symp., 1987, pp. 35-36.
73. Lloyd, D.K., Herrmann, C.R., and Corcoran, W.J.: "Techniques for Reliability Assessment and Growth in Large Solid Rocket Motor Development." AIAA Third Joint Propulsion Conference, paper No. 67-435, 1967.
74. Martz, H.F., and Waller, R.A.: "Bayesian Reliability Analysis." John Wiley and Sons, Inc., New York, 1982.
75. Mazzuchi, T.A., and Soyer, R.: "Reliability Assessment and Prediction During Product Development." Proc. Annual Reliability and Maintainability Symp., 1992, pp. 468-474.
76. Mazzuchi, T.A., and Soyer, R.: "A Bayesian Attribute Reliability Growth Model." Proc. Annual Reliability and Maintainability Symp., 1991, pp. 322-325.
77. Meth, M.A.: "Reliability Growth Myths and Methodologies: A Critical Review." Proc. Annual Reliability and Maintainability Symp., 1992, pp. 337-342.
78. Myhre, J.M., and Saunders, S.C.: "Comparison of Two Methods of Obtaining Approximate Confidence Intervals for System Reliability." Technometrics, vol. 10, February 1968, pp. 37-49.
79. Nelson, W.: "Applied Life Data Analysis." John Wiley and Sons, Inc., New York, 1982.
80. O'Connor, P.T.: "Practical Reliability Engineering." John Wiley and Sons, Inc., New York, 1991.

81. Olsen, D.E.: "A Confidence Interval for the Barlow-Scheuer Reliability Growth Model." IEEE Trans. Reliability, vol. R-27, April 1978, pp. 308-310.
82. Olsen, D.E.: "Estimating Reliability Growth." IEEE Trans. Reliability, vol. R-26, 1977, pp. 50-53.
83. Parr, V.B., and Webster, J.T.: "A Method for Discriminating Between Failure Density Functions Used in Reliability Prediction." Technometrics, vol. 7, February 1965, pp. 1-10.
84. Parzen, E.: "Stochastic Processes." Holden-Day, San Francisco, 1962.
85. Pollock, S.M.: "A Bayesian Reliability Growth Model." IEEE Trans. Reliability, vol. R-17, December 1968, pp. 187-198.
86. Prasad, P.S.K.: "Reliability Prediction and Growth Studies—How Accurate and Useful?" Microelectronics and Reliability, vol. 13, 1974, pp. 195-202.
87. Proschan, F.: "Reliability Growth." Proceedings of the 19th Conference on the Design of Experiments in Army Research, Development, and Testing, Rock Island, IL, October 1973.
88. Rabon, L.M.: "Application of Reliability Growth to Mechanical Systems." Proc. Annual Reliability and Maintainability Symp., 1982, pp. 439-443.
89. Read, R.R.: "A Remark on the Barlow-Scheuer Reliability Growth Estimation Scheme." Technometrics, vol. 13, 1971, pp. 199-200.
90. Robinson, D.G., and Dietrich, D.: "A New Nonparametric Growth Model." IEEE Trans. on reliability, vol. 36, 1987, pp. 411-418.
91. Robinson, D.G., and Dietrich, D.: "A System Level Reliability Growth Model." Proc. Annual Reliability and Maintainability Symp., 1988, pp. 243-247.
92. Safie, F.M.: "SSME Reliability Growth Study." Presentation MSFC, Reliability Analysis Branch, August 1990.
93. Safie, F.M., and Fox, E.P.: "A Probabilistic Design Analysis Approach for Launch Systems." AIEE/SAE/ASME 27th Joint Propulsion Conference, 1991.
94. Safie, F.M.: "Use of Probabilistic Design Methods for NASA Applications." Presentation ASME/WAM Symp. on Reliability Technology, Anaheim, CA, 1992.
95. Safie, F.M., and Taneja, V.S.: "Reliability Evaluation Methodology for NASA Space Transport Main Engine (STME)." Submitted for publication.
96. Sen, P.: "Optimization of Reliability Growth Problem." Microelectronics and Reliability, vol. 13, 1974, pp. 87-89.
97. Simkins, D.S.: "A Reliability Growth Management Approach." Proc. Annual Reliability and Maintainability Symp., 1979, pp. 356-360.

98. Singpurwalla, N.D.: "Time Series Analysis of Failure Date." Proc. Annual Reliability and Maintainability Symp., 1978, pp. 107–112.
99. Smith, A.F.M.: "A Bayesian Note on Reliability Growth During a Development Testing Program." IEEE Trans. Reliability, 1977, pp. 346–347.
100. Smith, S.A., and Oren, S.S.: "Reliability Growth of Repairable Systems." Naval Research Logistics Quarterly, vol. 27, December 1980, pp. 539–547.
101. Sriwastav, G.L.: "A Reliability Growth Model." IEEE Trans. Reliability, vol. R-27, 1978, pp. 306–307.
102. Stoll, C.N., and Oliveri, W.S.: "Reliability Growth: Actual Versus Predicted." Proc. Annual Reliability and Maintainability Symp., 1974, pp. 391–395.
103. Thompson, W.A. Jr.: "On the Foundations of Reliability." Technometrics, vol. 23, 1983, pp. 1–13.
104. Tung, S.S.: "A Reliability Growth Model." Proc. Annual Reliability and Maintainability Symp., 1984, pp. 490–492.
105. Virene, E.P.: "Reliability Growth and Its Upper Limit." Proc. Annual Symp. on Reliability, 1970, pp. 265–270.
106. Weinrich, M.C., and Gross, A.J.: "The Barlow-Schaver Reliability Growth Model From a Bayesian Viewpoint." Technometrics, vol. 20, August 1978, pp. 249–254.
107. Weiss, H.K.: "Estimation of Reliability Growth in a Complex System With Poisson-Type Failure." Operations Research, 1956, pp. 532–545.
108. Woods, W.M.: "The Effects of Discounting Failures and Weighting Data on the Accuracy of Some Reliability Growth Models." Proc. Annual Reliability and Maintainability Symp., 1990, pp. 200–209.
109. Wronka, J.C.: "Tracking of Reliability Growth in Early Development." Proc. Annual Reliability and Maintainability Symp., 1988, pp. 238–242.
110. Zelen, M.: "Statistical Theory of Reliability." University of Wisconsin Press, Madison, WI, 1963.
111. Ziad, T.: "Statistical Precision and Robustness of the AMSAA Continuous Reliability Growth Estimators." Proc. Annual Reliability and Maintainability Symp., 1990, pp. 210–214.

